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两多模泛函相干态叠加态的高次和压缩
——广义磁场分量的不等幂次高次和压缩效应

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摘 要： 利用多模压缩态理论，研究了多模泛函相干态 $|\{f_j(x,y,z)\}\rangle_q$ 及其相反态 $|\{-f_j(x,y,z)\}\rangle_q$ 的线性叠加所组成的两态叠加多模叠加态光场 $|\psi_{(f)}^{(2)}\rangle_q$ 的广义磁场分量的不等幂次高次和压缩特性。结果发现：1) 当各模的压缩次数 $N_j(j=1,2,\cdots,\cdots,q$ 下同) 之和为偶数时，态 $|\psi_{(f)}^{(2)}\rangle_q$ 恒处于不等幂次 N_j-H 最小测不准态；2) 当各模的压缩次数 N_j 之和为奇数时，态 $|\psi_{(f)}^{(2)}\rangle_q$ 在一定条件下可呈现出周期性变化的、任意不等幂次的广义非线性高次和压缩效应。

关键词： 量子光学；多模泛函相干态；两态叠加；多模压缩态；广义磁场分量；不等幂次高次和压缩

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1 引 言

光场非经典性研究是近年来量子光学领域内的一个十分活跃的前沿性的重大研究课题[1~7]。利用多模压缩态理论[1~4]，人们曾经对光场的压缩及高次压缩做过大量而又富有成效的研究工作，并且取得了许多重要成果[8~15]。但是所有这些研究，只是对组成态的多模相干态中的本征值取复常数进行探讨，而对其为泛函的情形尚未进行任何探讨。将多模相干态推广为多模泛函相干态，并使其本征值取空间坐标变量的泛函形式进行研究，不仅具有重要的学术价值，而且对于人们进一步探讨光场经典强度和经典振幅的任意空间分布对光场压缩特性的影响等具有重要的指导作用。

2 态 $|\psi_{(f)}^{(2)}\rangle_q$ 的数学结构

态 $|\psi_{(f)}^{(2)}\rangle_q$ 的数学表达式如下

$$|\psi_{(f)}^{(2)}\rangle_q = C_1^{(f)}|\{f_j(x,y,z)\}\rangle_q + C_2^{(f)}|\{-f_j(x,y,z)\}\rangle_q, \tag{1}$$

式中 $C_1^{(f)}$ 和 $C_2^{(f)}$ 为两态叠加的复几率幅，$f_j(x,y,z)$ 为一任意的复解析函数，其中

$$C_1^{(f)} = r_1^{(f)}\exp[\mathrm{i}\theta_1^{(f)}], \qquad C_2^{(f)} = r_2^{(f)}\exp[\mathrm{i}\theta_2^{(f)}], \tag{2}$$

$$f_j(x,y,z) = |f_j(x,y,z)|\exp[\mathrm{i}\varphi_j(x,y,z)] \qquad (j=1,2,3,\cdots,\cdots,q). \tag{3}$$

$|\{f_j(x,y,z)\}\rangle_q$ 和 $|\{-f_j(x,y,z)\}\rangle_q$ 分别为多模泛函相干态和其相反态，其表示如下

$$|\{f_j(x,y,z)\}\rangle_q = |f_1(x,y,z),f_2(x,y,z),\cdots,f_j(x,y,z),\cdots,f_{q-1}(x,y,z),f_q(x,y,z)\rangle =$$
$$\exp\left\{-\frac{1}{2}\left[\sum_{j=1}^{q}|f_j(x,y,z)|^2\right]\right\}\sum_{\{n_j=0\}}^{\infty}\left\{\prod_{j=1}^{q}\left[\frac{f_j^{n_j}(x,y,z)}{\sqrt{n_j!}}\right]\right\}|\{n_j\}\rangle_q, \tag{4}$$

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$$|\{-f_j(x,y,z)\}\rangle_q = |-f_1(x,y,z),-f_2(x,y,z),\cdots,-f_j(x,y,z),\cdots,-f_{q-1}(x,y,z),-f_q(x,y,z)\rangle = \exp\left\{-\frac{1}{2}\left[\sum_{j=1}^{q}|f_j(x,y,z)|^2\right]\right\}\sum_{\{n_j=0\}}^{\infty}\left\{\prod_{j=1}^{q}\left[\frac{(-f_j(x,y,z))^{n_j}}{\sqrt{n_j!}}\right]\right\}|\{n_j\}\rangle_q. \tag{5}$$

在以上各式中，q 为腔模总数，$|\{n_j\}\rangle_q = |n_1,n_2,\cdots,n_j,\cdots,n_q\rangle$ 为多模光子数态，$|f_j(x,y,z)|^2$ 为多模泛函相干态光场中第 j 模光场的平均光子数。因 $|f_j(x,y,z)|^2$ 与光场的经典强度成正比，所以，$|f_j(x,y,z)|^2$ 也就表征了第 j 模光场经典强度的空间分布情况。显见，第 j 模光场平均光子数的方根 $|f_j(x,y,z)|$ 与光场经典实振幅成正比，它可用以表征第 j 模光场经典实振幅的空间分布情况。而 $f_j(x,y,z)$ 则与第 j 模光场的经典复振幅成正比，(3) 式中的相位 $\varphi_j(x,y,z)$ 则可用以表征第 j 模光场经典相位的空间分布情况，而 $\sum_{j=1}^{q}|f_j(x,y,z)|^2$ 则为多模泛函相干态光场的总的平均光子数 (即总的经典强度) 的空间分布函数。

态 $|\psi_{(f)}^{(2)}\rangle_q$ 的归一化条件为

$${}_q\langle\psi_{(f)}^{(2)}|\psi_{(f)}^{(2)}\rangle_q = r_1^{(f)2} + r_2^{(f)2} + 2r_1^{(f)2}r_2^{(f)2}\cos\theta_1^{(f)} - \theta_2^{(f)})\exp\left[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\right] = 1. \tag{6}$$

3　一般理论结果

根据多模辐射光场的广义非线性不等幂次 N_j 次方 H 压缩即高次和压缩的定义，并利用本文的 (1)~(6) 式，经过计算，即可求得态 $|\psi_{(f)}^{(2)}\rangle_q$ 的广义磁场分量 (既第一正交相位分量) 的广义非线性不等幂次高次和压缩的一般理论结果如下

$$\begin{aligned}4\langle\Delta H_1^2(N_j)_q\rangle - \langle[B_q(N_j),B_q^+(N_j)]\rangle = {} & 2\prod_{j=1}^{q}|f_j(x,y,z)|^{2N_j}\left\{\left[r_1^{(f)2}+r_2^{(f)2}\right] + (-1)^{\sum_{j=1}^{q}N_j}\right. \\ & 2r_1^{(f)}r_2^{(f)}\cos(\theta_1^{(f)}-\theta_2^{(f)})\exp\left[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\right] + \cos\left\{2\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]\right\} - \\ & 2\left\{\left[r_1^{(f)2} + (-1)^{\sum_{j=1}^{q}N_j}r_2^{(f)2}\right]\cos\left\{\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]\right\} + \right. \\ & r_1^{(f)}r_2^{(f)}\left\{\cos\left\{\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)] + (\theta_1^{(f)}-\theta_2^{(f)})\right\} + (-1)^{\sum_{j=1}^{q}N_j}\right. \\ & \left.\left.\left.\cos\left\{\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)] - (\theta_1^{(f)}-\theta_2^{(f)})\right\}\right\}\exp\left[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\right]\right\}^2\right\}.\end{aligned} \tag{7}$$

式中 N_j 为多模光场中第 j 模光场的压缩次数，$\varphi_j(x,y,z)$ 为第 j 模光场经典初始相位，$(\theta_1^{(f)}-\theta_2^{(f)})$ 为态间的初始相位差。

4　不等幂次 $N_j - H$ 最小测不准态与不等幂次高次和压缩效应

4.1　各模压缩次数之和为偶数的情形

当 $\sum_{j=1}^{q}N_j = 2l(l=1,2,3,\cdots,\cdots)$ 时，(7) 式可进一步化为

$$4\langle\Delta H_1^2(N_j)_q\rangle - \langle[B_q(N_j),B_q^+(N_j)]\rangle = 0. \tag{8}$$

可见当各模的压缩次数之和为偶数时，态 $|\psi_{(f)}^{(2)}\rangle_q$ 可恒处于广义非线性不等幂次 N_j-H 最小测不准态，不呈现任何压缩效应。

4.2　各模压缩次数之和为奇数的情形

当 $\sum_{j=1}^{q}N_j = 2l+1(l=1,2,3,\cdots,\cdots)$ 时，(8) 式可进一步化为

$$4\langle\Delta H_1^2(N_j)_q\rangle-\langle[B_q(N_j),B_q^+(N_j)]\rangle=2\prod_{j=1}^{q}|f_j(x,y,z)|^{2N_j}\Big\{1-4r_1^{(f)}r_2^{(f)}\cos(\theta_1^{(f)}-\theta_2^{(f)})\times\exp\Big[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\Big]+\cos\Big\{2\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]\Big\}-2\Big\{[r_1^{(f)2}-r_2^{(f)2}]\times\cos\Big\{\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]\Big\}-2r_1^{(f)}-r_2^{(f)}\sin\Big\{\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]\Big\}\times\sin(\theta_1^{(f)}-\theta_2^{(f)})\exp\Big[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\Big]\Big\}^2\Big\}. \tag{9}$$

如果各模压缩次数与该模的经典初始相位乘积之和满足

$$\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]=n\pi+\pi/2\qquad(n\text{为整数}). \tag{10}$$

则 (9) 式可进一步化为

$$4\langle\Delta H_1^2(N_j)_q\rangle-\langle[B_q(N_j),B_q^+(N_j)]\rangle=-8\prod_{j=1}^{q}|f_j(x,y,z)|^{2N_j}r_1^{(f)}r_2^{(f)}\exp\Big[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\Big]\times\Big\{\cos(\theta_1^{(f)}-\theta_2^{(f)})+2r_1^{(f)}r_2^{(f)}\exp\Big[-2\sum_{j=1}^{q}|f_j(x,y,z)|^2\Big]\sin^2(\theta_1^{(f)}-\theta_2^{(f)})\Big\}. \tag{11}$$

如果态间初始相位差进一步满足条件

$$(\theta_1^{(f)}-\theta_2^{(f)})\in|2n\pi-\pi/2,2n\pi+\pi/2], \tag{12}$$

则

$$4\langle\Delta H_1^2(N_j)_q\rangle-\langle[B_q(N_j),B_q^+(N_j)]\rangle<0. \tag{13}$$

这就表明，在上述条件下，态 $|\psi_{(f)}^{(2)}\rangle$ 的广义磁场分量 (即第一正交相位分量) 存在广义非线性不等幂次高次和压缩效应。此外，由 (11) 式还可以看出：其压缩程度和压缩深度分别与两态叠加的几率幅 $r_1^{(f)}$ 和 $r_2^{(f)}$、压缩次数 N_j、腔模总数 q、第 j 模泛函相干态光场的经典强度 $|f_j(x,y,z)|^2$、经典振幅 $|f_j(x,y,z)|$、各模的经典初始相位 $\varphi_j(x,y,z)$、态间的初始相位差 $(\theta_1^{(f)}-\theta_2^{(f)})$、以及多模泛函相干态光场的总的平均光子数 (即总的经典强度) 的空间分布函数 $\sum_{j=1}^{q}|f_j(x,y,z)|^2$ 等呈很强的非线性关联。

如图所示。其中图 1 给出了 $q=2$、$N_1=3$、$N_2=4$、$r_1^{(f)}=r_2^{(f)}$、$(\theta_1^{(f)}-\theta_2^{(f)})=\pi/4$、$\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]=\pi/2$ 情况下两正交相位分量的压缩随 $|f_j(x,y,z)|(x$ 表示) 和 $|f_2(x,y,z)|(y$ 表示) 的变化情况。图 2 给出了 $q=3$、$r_1^{(f)}=r_2^{(f)}$、各模压缩次数 $N_j=3$ 和 $|f_j(x,y,z)|=0.5(j=1,2,3)$ 时两正交相

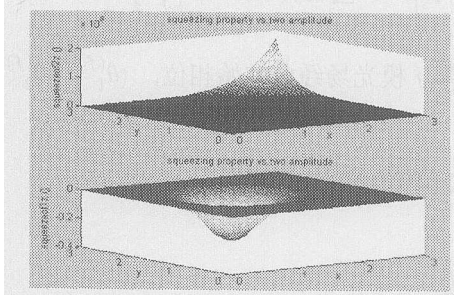

Fig.1 The squeezing of two perpendicular phase components vs two real amplitudes of $|f_1(x,y,z)|$ and $|f_2(x,y,z)|$

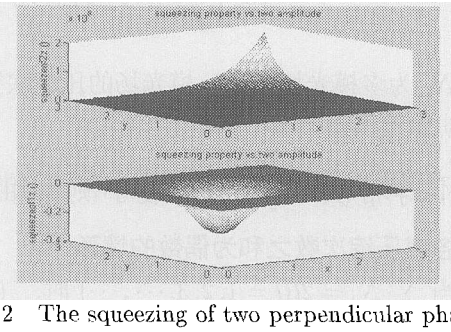

Fig.2 The squeezing of two perpendicular phase components vs two phases of $(\theta_1^{(f)}-\theta_2^{(f)})$ and $\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]$

位分量的压缩随 $(\theta_1^{(f)}-\theta_2^{(f)})(x$ 表示) 和 $\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]$ (y 表示) 的变化情况。为便于比较，我们在每幅图中均给出了广义磁场和广义电场这两个正交相位分量的压缩情况。其中，每幅图的上部表示广义电场分量，下部表示广义磁场分量。

5 结　论

综上所述，可得以下 4 点结论：

1) 当各模压缩次数之和 $\sum_{j=1}^{q} N_j$ 为偶数时，态 $|\psi_{(f)}^{(2)}\rangle_q$ 恒处于 $N_j - H$ 最小测不准态。不存在压缩效应。

2) 当各模压缩次数之和 $\sum_{j=1}^{q} N_j$ 为奇数时，如果各模的压缩次数与该模的经典初始相位乘积之和 $\sum_{j=1}^{q}[N_j\varphi_j(x,y,z)]$ 的取值满足式 (10)、态间的初始相位差 $(\theta_1^{(f)} - \theta_2^{(f)})$ 满足式 (12) 时，态 $|\psi_{(f)}^{(2)}\rangle_q$ 的广义磁场分量存在广义非线性不等幂次高次和压缩效应。

3) 态 $|\psi_{(f)}^{(2)}\rangle_q$ 的不等幂次高次和压缩效应的压缩程度和深度与两态叠加的几率幅 $r_1^{(f)}$ 和 $r_2^{(f)}$、压缩次数 N_j、腔模总数 q、第 j 模泛函相干态光场的经典振幅 $|f_j(x,y,z)|$ 和经典强度 $|f_j(x,y,z)|^2$、各模的经典初始相位 $\varphi_j(x,y,z)$、态间的经典初始相位差 $(\theta_1^{(f)} - \theta_2^{(f)})$、以及多模泛函相干态光场的总的经典强度 $\sum_{j=1}^{q} f_j(x,y,z)|^2$ 等呈很强的非线性关联。

4) 由图 1 和图 2 可以看出，态 $|\psi_{(f)}^{(2)}\rangle_q$ 的广义磁场分量和广义电场分量所分别呈现的广义非线性不等幂次高次和压缩效应，存在着周期性的互补压缩关系。

参考文献：

[1] Yang Zhiyong, Hou Xun. Two kinds of nonlinear higher-order squeezing effects in a new type of two-mode light field of superposition state [J]. *Acta Photonica Sinica* (光子学报), 1998, 27(4): 289-299 (in Chinese).

[2] Hou Xun, Yang Zhiyong. Properties of nonlinear higher-order squeezing effects in a new type of multi-mode light field state of superposition state with distinguishable two quantum states in the kind (I) [J]. *Acta Photonica Sinica* (光子学报), 1998, 27(10): 865-878 (in Chinese).

[3] Yang Zhiyong, Hou Xun. Nonlinear higher-order difference squeezing of multi-mode radiative light field-general theory of N-th X-squeezing effects [J]. *Acta Photonica Sinica* (光子学报), 1998, 27(12): 1065-1069 (in Chinese).

[4] Yang Zhiyong, Hou Xun. General theory on generalized nonlinear unequal-order higher-order squeezing of multi-mode radiative light field [J]. *Acta Photonica Sinica* (光子学报), 1999, 14(5): 15-20 (in Chinese).

[5] Yang Zhiyong, Hou Xun. Properties of nonlinear higher-order squeezing in second kind of multi-mode light field state of superposition state with distinguishable two quantum states [J]. *Acta Photonica Sinica* (光子学报), 1998, 27(11): 962-973 (in Chinese).

[6] Zhang Z M, Lei Xu, *et al.* A new kind of higher-order squeezing of radiation field [J]. *Phys. Lett. (A)*, 1990, 150(1): 27-30.

[7] Hou Xun, Yang Zhingyong, Xu Dingguo, *et al.* Study on the properties of both generalized nonlinear equal-ordern-th powery-squeezing and generalized nonlinear equal-order N-th power H-squeezing in the third and the fourth kinds of multimode superposition state light field with the superpositions of macroscopically distinct two quantum states –on the phenomena of similitude squeezing and degenerate squeezing [J]. *Acta Photonica Sinica* (光子学报), 2000, 29(5): 385-395 (in Chinese).

[8] Wang Juxia, Yang Zhiyong, Hou Xun, *et al.* Properties of the second-order unequal-power N_j-th power H-squeezing in the two new type of multi-mode superposition state light field [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2002, 19(5): 450-456 (in Chinese).

[9] Han Xiaowei, Yang Zhiyong, Hou Xun. Study on the higher-power H-squeezing properties of in the second kind of multi-mode superposition light field [J]. *Chinese Journal of Quantum Electronics* (量子电子学报), 2002, 19(4): 343-346 (in Chinese).

[10] Yang Zhiyong, Cao Dagang, Wan Huijun, *et al.* The equal-power higher-power sum squeezing of generalized magnetic field component in the first kind of three state superposition multimode superposition state field [J]. *Acta Photonica Sinica* (光子学报), 2002, 31(9): 1041-1046 (in Chinese).

[11] Yang Zhiyong, An Yuying, Miao Runcai, *et al.* Higher-power sum squeezing of generalized electric field component in the first field with superposition state light field with superposition of macroscopically distinct three quantum states [J]. *Journal of Shanxi Normal Univercity (Natural Science Edition)* (陕西师范大学学报 (自然科学版)), 2003, 31(1): 48-54 (in Chinese).

[12] Yang Zhiyong, Sun Zhongyu, Hu Yangfang, *et al.* Generalized nonlinear equal-power $(2m+1)$-th power Y-squeezing in the second kind of multimode superposition state light field with superpositions of macroscopically distinct three quantum states [J]. *Acta Photonica Sinica* (光子学报), 2002, 31(7): 785-790 (in Chinese).

[13] Sun Zhongyu, Chen Guangde, Yang Zhiyong, *et al.* Effects of equal-power N-th Power Y-squeezing of generalized electric field component in the state $\psi_3^{(3)}\rangle_q$ [J]. *Laser Journal* (激光杂志), 2003, 24(2): 35-37 (in Chinese).

[14] Sun Zhongyu, Chen Guangde, Yang Zhiyong, *et al.* The properties of unequal-power N_j-th Power Y-squeezing in the third kind of multimode superposition state light field with the superposition of three quantum states [J]. *Journal of Optoelectronics • Laser* (光电子 • 激光), 2003, 14(2): 201-205 (in Chinese).

[15] Xu Dingguo, An Yuying, Xia Congling. Equal-order N-th order H-squeezing of generalized magnetic field component in multi-mode superposition state light field [J]. *Journal of Shanxi Normal Univercity (Natural Science Edition)* (陕西师范大学学报 (自然科学版)), 2003, 31(2): 55-59 (in Chinese).

Effects of generalized nonlinear unequal-power higher-power sum-squeezing of superposition state light-field composed of the two different multimode functional coherent states

—unequal higher power sum-squeezing of generalized magnetic-field component

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Abstract: In this paper, the multimode squeezed state theory is utilized to study the characteristics of generalized nonlinear-unequal-power higher-power sum-squeezing of generalized magnetic-field component in the state $|\psi_{(f)}^{(2)}\rangle_q$ made up of the linear superposition of multimode functional coherent state $|\{f_j(x,y,z)\}\rangle_q$ and its contrary state $|\{-f_j(x,y,z)\}\rangle_q$ according to the superposition principle of quantum state in quantum mechanics. It is found that: 1) under the condition of the sum of squeezing-power-number being an any even-number, the state $|\psi_{(f)}^{(2)}\rangle_q$ can always stay in the generalized nonlinear unequal-power N_j-H minimum uncertainty state;and 2) under the condition of the sum of squeezing-power-number being an any odd-number, while some other fixed conditions are satisfied by the state $|\psi_{(f)}^{(2)}\rangle_q$, the state mentioned can display the effects of generalized nonlinear unequal-power N_j-th power H-squeezing that changes periodically and alternatively.

Key words: quantum optics; multimode functional coherent state; two state superposition; multimode squeezed state; generalized magnetic-field component; unequal-power N_j-th power H-squeezing

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